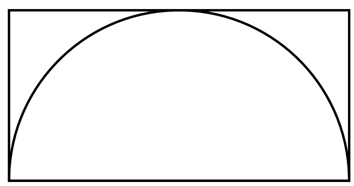
Time limit: 90 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Arjun eats twice as many chocolates as Theo, and Wen eats twice as many chocolates as Arjun. If Arjun eats 6 chocolates, compute the total number of chocolates that Arjun, Theo, and Wen eat.

2. Compute $1 \times 4 - 2 \times 3 + 2 \times 5 - 3 \times 4 + 3 \times 6 - 4 \times 5 + 4 \times 7 - 5 \times 6 + 5 \times 8 - 6 \times 7$.

3. A semicircle of radius 2 is inscribed inside of a rectangle, as shown in the diagram below. The diameter of the semicircle coincides with the bottom side of the rectangle, and the semicircle is tangent to the rectangle at all points of intersection. Compute the length of the diagonal of the rectangle.



4. Suppose a, b, and c are numbers satisfying the three equations:

$$a + 2b = 20,$$
$$b + 2c = 2,$$
$$c + 2a = 3.$$

   Find $9a + 9b + 9c$.

5. Lakshay chooses two numbers, m and n, and draws two lines, $y = mx + 3$ and $y = nx + 23$. Given that the two lines intersect at $(20, 23)$, compute $m + n$.

6. Compute the three-digit number that satisfies the following properties:
   - The hundreds digit and ones digit are the same, but the tens digit is different.
   - The number is divisible by 9.
   - When the number is divided by 5, the remainder is 1.

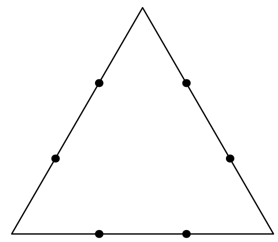
7. Recall that an arithmetic sequence is a sequence of numbers such that the difference between any two consecutive terms is the same. Suppose x_1, x_2, x_3 forms an arithmetic sequence. If $x_2 = 2023$, compute $x_1 + x_2 + x_3$.

8. One of Landau's four unsolved problems asks whether there are infinitely many primes p such that $p - 1$ is a perfect square. How many such primes are there less than 100?

9. The boxes in the expression below are filled with the numbers $3, 4, 5, 6, 7$, and 8, so that each number is used exactly once. What is the least possible value of the expression?

$$\square \times \square + \square \times \square - \square \times \square$$

10. Consider an equilateral triangle with side length 9. Each side is divided into 3 equal segments by 2 points, for a total of 6 points. Compute the area of the circle passing through these 6 points.



11. Consider two geometric sequences $16, a_1, a_2, \ldots$ and $56, b_1, b_2, \ldots$ with the same common nonzero ratio. Given that $a_{2023} = b_{2020}$, compute $b_6 - a_6$.

12. Find the greatest integer less than $\sqrt{10} + \sqrt{80}$.

13. Three people, Pranav, Sumith, and Jacklyn, are attending a fair. Every time a person enters or exits, the groundskeeper writes their name down in chronological order. If each person enters and exits the fairgrounds exactly once, in how many ways can the groundskeeper write down their names?

14. For real numbers x and y, suppose that $|x| - |y| = 20$ and $|x| + |y| = 23$. Compute the sum of all possible distinct values of $|x - y|$.

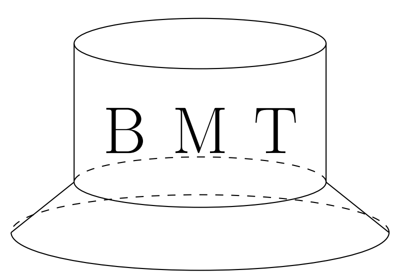
15. Find the number of positive integers n less than 10000 such that there are more 4's in the digits of $n + 1$ than in the digits of n.

16. Let n be the smallest positive integer such that there exist integers, a, b, and c, satisfying:

$$\frac{n}{2} = a^2, \quad \frac{n}{3} = b^3, \quad \frac{n}{5} = c^5.$$

Find the number of positive integer factors of n.

17. Jingyuan is designing a bucket hat for BMT merchandise. The hat has the shape of a cylinder on top of a truncated cone, as shown in the diagram below. The cylinder has radius 9 and height 12. The truncated cone has base radius 15 and height 4, and its top radius is the same as the cylinder's radius. Compute the total volume of this bucket hat.



18. Kait rolls a fair 6-sided die until she rolls a 6. If she rolls a 6 on the Nth roll, she then rolls the die N more times. What is the probability that she rolls a 6 during these next N times?

19. Let ω be a circle with center O and radius 8, and let A be a point such that $AO = 17$. Let P and Q be points on ω such that line segments $\overline{AP}$ and $\overline{AQ}$ are tangent to ω. Let B and C be points chosen on $\overline{AP}$ and $\overline{AQ}$, respectively, such that $\overline{BC}$ is also tangent to ω. Compute the perimeter of triangle $\triangle ABC$.

20. Call a positive integer, n, *ready* if all positive integer divisors of n have a ones digit of either 1 or 3. Let S be the sum of all positive integer divisors of 32! that are ready. Compute the remainder when S is divided by 131.

21. Let p, q, and r be the three roots of the polynomial $x^3 - 2x^2 + 3x - 2023$. Suppose that the polynomial $x^3 + Bx^2 + Mx + T$ has roots $p+q$, $p+r$, and $q+r$ for real numbers B, M, and T. Compute $B - M + T$.

22. Triangle $\triangle ABC$ has side lengths $AB = 8$, $BC = 15$, and $CA = 17$. Circles ω_1 and ω_2 are externally tangent to each other and within $\triangle ABC$. The radius of circle ω_2 is four times the radius of circle ω_1. Circle ω_1 is tangent to $\overline{AB}$ and $\overline{BC}$, and circle ω_2 is tangent to $\overline{BC}$ and $\overline{CA}$. Compute the radius of circle ω_1.

23. Let N be the number of positive integers x less than $210 \cdot 2023$ such that

$$\operatorname{lcm}(\gcd(x, 1734), \gcd(x + 17, x + 1732))$$

divides 2023. Compute the sum of the prime factors of N with multiplicity. (For example, if $S = 75 = 3^1 \cdot 5^2$, then the answer is $1 \cdot 3 + 2 \cdot 5 = 13$).

24. Define a sequence $a_0, a_1, a_2, \ldots$ recursively by $a_0 = 0$, $a_1 = 1$, and $a_{n+2} = a_{n+1} + xa_n$ for each $n \geq 0$ and some real number x. The infinite series

$$\sum_{n=0}^{\infty} \frac{a_n}{10^n} = 1.$$

Compute x.

25. A tetrahedron has three edges of length 2 and three edges of length 4, and one of its faces is an equilateral triangle. Compute the radius of the sphere that is tangent to every edge of this tetrahedron.