Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Find $f'\left(\frac{\pi}{4}\right)$, where $f(x) = 20\cos(x) + 23\sin(x)$.

2. Compute
$$\int_0^4 (x-2)^5 + (x-2)^6 + (x-2)^7 \, \mathrm{d}x\,.$$

3. Let A be the area of the region bounded by $x = 0$, $y = 0$, $x = 6$, and $y = \sqrt{kx}$, for some real number $k > 0$. If $A = 36$, compute the value of k.

4. An ice cube melts such that it always remains a cube, and its volume decreases at a constant rate. The initial side length of the cube is 10 inches, and it takes 50 minutes for the ice cube to completely melt. When the side length of the ice cube is 4 inches, what is the rate, in inches per minute, at which the side length of the ice cube is decreasing?

5. Let $f(a, b) = b^3 - a^3 + a^2 b - ab^2$. There exists a real number C such that regardless of the choice of nonnegative real numbers $0 = x_0 < x_1 < x_2 < x_3 < \cdots < x_n = 4$, we have $C \le \sum_{i=1}^{n} f(x_{i-1}, x_i)$. Compute the maximum value of C.

6. For a positive number x, let $f_0(x) = \frac{1}{x}$ and $f_n(x) = \frac{d^n}{dx^n}\left(\frac{1}{x}\right)$ for all positive integers n. If
$$g(x) = \sum_{n=0}^{\infty} \frac{1}{f_n(x)},$$
compute $g(1)$.

7. Define a sequence $a_0, a_1, a_2, \ldots$ by $a_0 = 24$, $a_1 = 23$, and $a_{n+2} = -a_{n+1} + 6a_n$ for $n \ge 0$. Compute
$$\sum_{n=1}^{\infty} \frac{a_n}{n 6^n}.$$

8. Let $f : [1, \infty) \to \mathbb{R}$ be a continuous function such that
$$I(f) = \int_1^{\infty} \left(\sqrt{2023}\, x e^{-x} f(x) - \frac{1}{4} x^2 f(x)^2\right) \mathrm{d}x$$
converges and is maximized over all continuous functions on $[1, \infty) \to \mathbb{R}$. Compute $f(1) + I(f)$.

9. Compute
$$\int_0^{2\pi} (\sin(x) + \cos(x))^6 \, \mathrm{d}x\,.$$

10. Compute
$$\int_0^{\infty} \frac{\sin(x)}{x^2} \sum_{n=1}^{\infty} \frac{\sin(nx)}{n!} \, \mathrm{d}x\,.$$