1. A particle starts moving from $(20, 0)$ along the line $x = 20$ at a speed of 1 unit per second in the positive y direction. Let $f(t)$ be the distance of the particle from the origin at time t - therefore, $f(20) = 20\sqrt{2}$. Then $f'(20)$ can be written in the form $\frac{p\sqrt{q}}{r}$, where p, q, and r are positive integers such that p and r are relatively prime and that q is square-free. Compute $p + q + r$.

   Answer: 5

   Solution: $f(t) = \sqrt{t^2 + 400}$. Therefore, $f'(t) = \frac{t}{\sqrt{t^2+400}}$. Therefore, $f'(20) = \frac{20}{\sqrt{800}} = \frac{\sqrt{2}}{2}$ and our answer is $\boxed{5}$.

2. For all real numbers x, let $f(x) = \left|x^2 + x\right|$. Let $I_1 = \displaystyle\int_{-2020}^{0} f(x)\ \mathrm{d}x$, and let $I_2 = \displaystyle\int_{0}^{2019} f(x)\ \mathrm{d}x$. Then $|I_1 - I_2|$ can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

   Answer: 7

   Solution: Observe that

   - For $x \le -1$, $f(-x-1) = \left|(-x-1)^2 + (-x-1)\right| = x^2 + x$.
   - For $-1 \le x \le 0$, $f(x) = -(x^2 + x)$.
   - For $x \ge 0$, $f(x) = x^2 + x$.

   Thus we can write

$$\begin{aligned} I_1 &= \int_{-2020}^{-1} \left(x^2 + x\right) \mathrm{d}x + \int_{-1}^{0} -\left(x^2 + x\right) \mathrm{d}x \\ &= \int_{0}^{2019} \left(x^2 + x\right) \mathrm{d}x + \frac{1}{6} \\ &= I_2 + \frac{1}{6} \\ I_1 - I_2 &= \frac{1}{6}. \end{aligned}$$

   Our answer, therefore, is $\boxed{7}$.

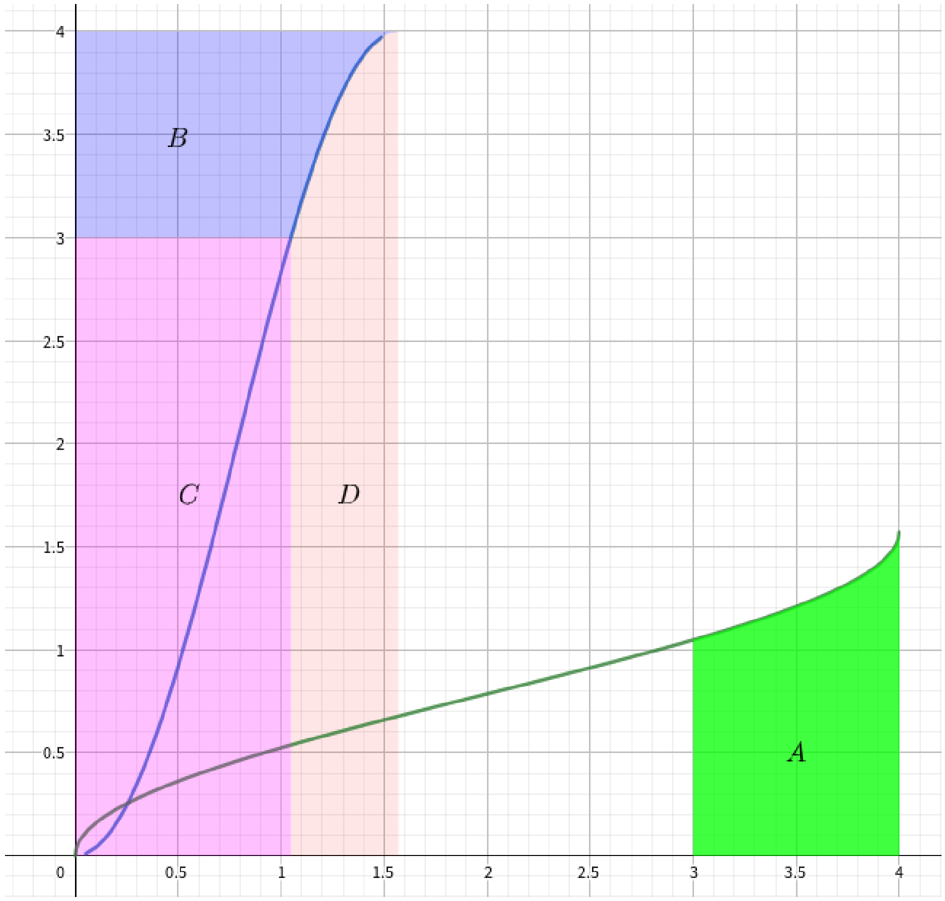
3. The integral

$$\int_3^4 \arcsin\left(\frac{\sqrt{x}}{2}\right) \mathrm{d}x$$

   can be written in the form $\frac{m\pi}{n} - \frac{p\sqrt{q}}{r}$, where m and n are relatively prime positive integers, p and r are relatively prime positive integers, and q is a square-free positive integer. Compute $m + n + p + q + r$.

   Answer: 11

   Solution: Refer to the diagram for labeling. First define $f(x) = \arcsin\left(\frac{\sqrt{x}}{2}\right)$, then by simple algebra $f^{-1}(x) = 4\sin^2(x)$. Then the integral, which equals to the area of A in the diagram, is also equal to the area of B (by simple reflection over the line $y = x$). Note that the green curve is $f(x)$ and the blue curve is $f^{-1}(x)$. Then B is the region we want the area of, C is the



rectangle $[0, \frac{\pi}{3}] \times [0, 3]$, D is the region under the curve $y = f^{-1}(x)$ in the interval $[\frac{\pi}{3}, \frac{\pi}{2}]$, and $B \cup C \cup D$ is the rectangle $[0, \frac{\pi}{2}] \times [0, 4]$. Then

$$\begin{aligned}
|B| &= |B \cup C \cup D| - |C| - |D| \\
&= \left(4 \cdot \frac{\pi}{2}\right) - \left(\frac{\pi}{3} \cdot 3\right) - \left(\int_{\pi/3}^{\pi/2} 4\sin^2(x)\,\mathrm{d}x\right) \\
&= 2\pi - \pi - \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right) \\
&= \frac{2\pi}{3} - \frac{\sqrt{3}}{2},
\end{aligned}$$

where the integral is computed by a standard integration by parts, or other trigonometric identities. Therefore, our answer is $\boxed{11}$.