Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers are positive integers. Only submitted answers will be considered for grading.
No calculators.

1. Let
$$f(x) = \frac{x^{2020}}{2020} + 2020!.$$
Compute $f''(1)$.

2. Compute the integral
$$\int_{-20}^{20} (20 - |x|)\,\mathrm{d}x\,.$$

3. Suppose $f\colon \mathbb{R} \to \mathbb{R}$ is a differentiable function defined by
$$f(x) = f'(2)x^2 + x.$$
Then $f(2)$ can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

4. If a is a positive real number such that the region of finite area bounded by the curve $y = x^2 + 2020$, the line tangent to that curve at $x = a$, and the y-axis has area 2020, compute a^3.

5. Suppose that a parallelogram has a vertex at the origin of the 2-dimensional plane, and two of its sides are vectors from the origin to the points $(10, y)$, and $(x, 10)$, where $x, y \in [0, 10]$ are chosen uniformly at random. The probability that the parallelogram's area is at least 50 is $\ln(\sqrt{a}) + \frac{b}{c}$, where a, b, and c are positive integers such that b and c are relatively prime and a is as small as possible. Compute $a + b + c$.

6. For some $a > 1$, the curves $y = a^x$ and $y = \log_a(x)$ are tangent to each other at exactly one point. Compute $|\ln(\ln(a))|$.

7. The limit
$$\lim_{n\to\infty} n^2 \int_0^{1/n} x^{x+1}\,\mathrm{d}x$$
can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

8. The summation
$$\sum_{a=1}^{\infty}\sum_{b=1}^{\infty} \frac{1}{a^2b + 2ab + ab^2}$$
can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

9. Let $f\colon \mathbb{R}_{>0} \to \mathbb{R}$ (where $\mathbb{R}_{>0}$ is the set of all positive real numbers) be differentiable and satisfy the equation
$$f(y) - f(x) = \frac{x^x}{y^y} f\left(\frac{y^y}{x^x}\right)$$
for all real $x, y > 0$. Furthermore, $f'(1) = 1$. Compute $\frac{f(2020^2)}{f(2020)}$.

10. The integral

$$\int_0^{\frac{\pi}{2}} \frac{x}{\tan(x)}\,\mathrm{d}x$$

can be written in the form $a^b \pi \ln c$, where a, b, and c are integers such that c is as small as possible. Compute $a + b + c$.