1. What is the sum of all positive integers less than 30 divisible by 2, 3, or 5?

2. Define $a \star b$ to be $2ab + a + b$. What is $((3 \star 4) \star 5) - (4 \star (5 \star 3))$?

3. Consider an equilateral triangle and square, both with are 1. What is the product of their perimeters?

4. Let ABC be have side lengths $3, 4$, and 5. Let P be a point in side ABC. What is the minimum sum of lengths of the altitudes from P to the side lengths of ABC?

5. Positive integers x, y, z satisfy $(x + yi)^2 - 46i = z$. What is $x + y + z$?

6. Let $g_0 = 1, g_1 = 2, g_2 = 3$, and $g_n = g_{n-1} + 2g_{n-2} + 3g_{n-3}$. For how many $0 \leq i \leq 100$ is it that g_i is divisible by 5?

7. Define $P(\tau) = (\tau + 1)^3$. If $x + y = 0$, what is the minimum possible value of $P(x) + P(y)$?

8. Simplify $\dfrac{1}{\sqrt[3]{81} + \sqrt[3]{72} + \sqrt[3]{64}}$

9. On 5×5 grid of lattice points, every point is uniformly randomly colored blue, red, or green. Find the expected number of monochromatic triangles T with vertices chosen from the lattice grid, such that some two sides of T are parallel to the axis.

10. Define $T_n = \displaystyle\sum_{i=1}^{n} i(n + 1 - i)$. Find
$$\lim_{n \to \infty} \frac{T_n}{n^3}$$

11. The roots of the polynomial $x^3 - \frac{3}{2}x^2 - \frac{1}{4}x + \frac{3}{8} = 0$ are in arithmetic progression. What are they?

12. What is the number of nondecreasing positive integer sequences of length 7 whose last term is at most 9?

13. The quartic equation $y = x^4 + 2x^3 - 20x^2 + 8x + 64$ contains the points $(-6, 160)$, $(-3, -113)$ and $(2, 32)$. A cubic $y = ax^3 + bx + c$ also contains these points. Determine the x-coordinate of the fourth intersection of the cubic with the quartic.

14. Three circles of radius 1 are inscribed in a square of side length s, such that the circles do not overlap or coincide with each other. What is the minimum s where such a configuration is possible?

15. How many ways can we pick four 3-element subsets of $\{1, 2, ..., 6\}$ so that each pair of subsets share exactly one element?

16. What is the radius of the largest sphere that fits inside the tetrahedron whose vertices are the points $(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1)$?

17. Consider triangle ABC in xy-plane where A is at the origin, B lies on the positive $x-$axis, C is on the upper right quadrant, and $A = 30^\circ, B = 60^\circ, C = 90^\circ$. Let the length $BC = 1$. Draw the angle bisector ℓ of angle C, and let this intersect the y-axis at D. What is the area of quadrilateral $ADBC$?

18. Define r_n to be the number of integer solutions to $x^2+y^2 = n$. Determine $\lim_{n\to\infty} \frac{r_1 + r_2 ... + r_n}{n}$.

19. Regular tetrahedron $P_1P_2P_3P_4$ has side length 1. Define P_i for $i > 4$ to be the centroid of tetrahedron $P_{i-1}P_{i-2}P_{i-3}P_{i-4}$, and $P_\infty = \lim_{n\to\infty} P_n$. What is the length of P_5P_∞?

20. Find
$$\prod_{k=1}^{2017} e^{\pi ik/2017} 2\cos\left(\frac{\pi k}{2017}\right)$$