1. Note that $2^{-2}+2^{-3}=\frac{1}{4}+\frac{1}{8}=0.375$. In addition, $2^4+2^3=16+8=24$. Hence the final answer is $(-2)(-3)(4)(3)=\boxed{72}$

2. We have that

$$\frac{1}{1-x}=1+x+x^2+x^3+x^4\dots$$
$$\left(\frac{1}{1-x}\right)'=\frac{1}{(1-x)^2}=1+2x+3x^2+4x^3+\dots=g(x)$$
$$g(x)=\sum_{k=0}^{\infty}(k+1)x^k$$

Differentiating again we have

$$g(x)'=\sum_{k=0}^{\infty}(k+1)kx^{k-1}=\left(\frac{1}{(1-x)^2}\right)'=\frac{2}{(1-x)^3}=\frac{2g(x)}{1-x}=2f(x)$$
$$\sum_{k=0}^{\infty}(k+1)kx^{k-1}=2f(x)$$
$$\sum_{k=0}^{\infty}\frac{(k+1)k}{2}x^{k-1}=f(x)$$

Hence the coefficent of x^{2015} will be $\frac{(2017)(2016)}{2}=\boxed{\binom{2017}{2}}$

3. We add the first two equations to get:

$$x^2+2y^2+3z^2=36 \quad (1)$$
$$+3x^2+2y^2+z^2=84 \quad (2)$$
$$4x^2+4y^2+4z^2=120 \quad (3)$$
$$x^2+y^2+z^2=30 \quad (4)$$

We can now put the above equation into equation 1 and 2 to get $x^2-z^2=24$. Now adding two times equation 4 and two times $xy+xz+yz$ we have:

$$2(x^2+y^2+z^2)+2(xy+xz+yz)=$$
$$(x+z)^2+(x+y)^2+(y+z)^2=2(30)+2(-7)=46.$$

From here it is easy to check for integer solutions by plugging in the squares from 1 to 6 and then checking whether the remaining number can be expressed as the sum of two squares. (It might be helpful to know that a number can be expressed as a sum of two squares if and only if its factorization into distinct primes contains no odd powers of primes congruent to 3 modulo 4). We then see that the only solution that works is $6^2+3^2+1^1=36+9+1=46$. Combining this with $x^2-z^2=24$ we get that the only integer solutions are $\boxed{(5,-2,1),(-5,2,-1)}$

4. From the recurrence relation we have $a_n=a_{n+1}-a_{n+2}=a_{n+1}-(a_{n+1}+a_{n+3})=-a_{n+3}=a_{n+6}$. Hence the sequence cycles with period 6. Writing out the first few terms and noting that $2015\equiv 5 \mod 6$, we get that $a_1+a_2+a_3+\dots+a_{2015}=a_1+a_2+a_3+a_4+a_5=-1+2+3+1-2=\boxed{3}$

5. Completing the square we get $x^2 - 4x + y^2 + 3 = (x-2)^2 + y^2 = 1$. From here we see that $x^2 + y^2$ would be the square of distance from the orgin to a point on the circle $(x-2)^2 + y^2 = 1$. The maximum and minimum distance would then be 3 and 1 respectively so our answer is $9 - 1 = \boxed{8}$

6. Let the five roots be $a, ar, ar^2, ar^3, ar^4, ar^5$. We are then given that:

$$a(1 + r + r^2 + r^3 + r^4) = 180 \tag{5}$$
$$-a^5 r^{10} = D \tag{6}$$
$$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4} = 20 \tag{7}$$

from Vieta's formulas. Now simplifying equation 7 we have:

$$\begin{aligned} \frac{1 + r + r^2 + r^3 + r^4}{ar^4} &= 20 \\ \frac{180}{a^2 r^4} &= 20 \\ (ar^2)^2 &= 9 \\ ar^2 &= \pm 3 \end{aligned}$$

Hence $D = -(ar^2)^5 = -(\pm 3)^5 = \pm 243 \implies |D| = \boxed{243}$

7. By the binomial theorem we have $(1+x)^{75} = \sum_{k=0}^{75} \binom{75}{k}$ Plugging in i we see that that our sum $S = \sum_{k=0}^{37} (-1)^k \binom{75}{2k}$ would be the real part of $(1+i)^{75}$. Hence converting to $1+i$ to polar form we have

$$(1+i)^{75} = (\sqrt{2}, \pi/4)^{75} = \left(2^{75/2}, \frac{75\pi}{4}\right) = \left(2^{75/2}, \frac{3\pi}{4}\right)$$

Computing the real part we have $S = Re\left(2^{75/2}, \frac{3\pi}{4}\right) = 2^{75/2} \cos\frac{3\pi}{4} = \boxed{-2^{37}}$

8. Let $x_k = \omega^k$. Then

$$P = \prod_{k=0}^{6} (1 + x_k - x_k^2) = -\prod_{k=0}^{6} (\phi - x_k)(\tau - x_k)$$

where ϕ, τ are the two solutions to $x^2 - x - 1$. Since ω is a primitive root of unity, as we go over all powers we pick up all the roots of unity. Hence we must have that $P = -(\phi - 1)(\phi - \omega)(\phi - \omega^2)\ldots(\phi - \omega^6)(\tau - 1)(\tau - \omega)(\tau - \omega^2)\ldots(\tau - \omega^6) = -(\phi^7 - 1)(\tau^7 - 1)$. Since both ϕ, τ satisfy $x^2 = x + 1$, we have that

$$\begin{aligned} x^7 - 1 = x * x^6 - 1 &= x(x+1)^3 - 1 \\ &= x(x^3 + 3x^2 + 3x + 1) - 1 \\ &= x^2(x^2 + 3x + 3) + x - 1 \\ &= (x+1)(4x+4) + x - 1 \\ &= 4x^2 + 9x + 3 \\ &= 13x + 7 \end{aligned}$$

Now since $\phi + \tau = 1, \phi\tau = -1$, we have that

$$P = -(13\phi + 7)(13\tau + 7) = -(-169 + 91 + 49) = \boxed{29}$$

9. [1] We wish to find

$$L = \lim_{n\to\infty} \frac{1}{n^2}\left(\sqrt{n^2-1} + \sqrt{n^2-2^2} + \ldots + \sqrt{n^2-(n-1)^2}\right)$$

Bringing a factor of $1/n$ into the square roots we have

$$\begin{aligned} L &= \lim_{n\to\infty} \frac{1}{n}\left(\sqrt{1-\frac{1}{n^2}} + \sqrt{1-\frac{2^2}{n^2}} + \ldots + \sqrt{1-\frac{(n-1)^2}{n^2}}\right) \\ &= \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n}\sqrt{1-\left(\frac{k}{n}\right)^2} \end{aligned}$$

Notice this is a Riemann sum with $\Delta = \dfrac{1}{n}$ and $x_k^* = \dfrac{k}{n}$. In the limit as n goes to infinity this converges to the integral $\displaystyle\int_0^1 \sqrt{1-x^2}\ \mathrm{d}x$. This is just one quarter of the area of the unit circle so the final answer is $\boxed{\dfrac{\pi}{4}}$

10. We have that $I = \displaystyle\int_0^{\pi/2} \ln(4\sin x)\ \mathrm{d}x = \int_0^{\pi/2} \ln(4\cos x)\ \mathrm{d}x$ since $\sin x$ and $\cos x$ take on the same values on the interval $[0, \pi/2]$. Adding these we have

$$\begin{aligned} 2I &= \int_0^{\pi/2} \ln(4\sin x)\ \mathrm{d}x + \int_0^{\pi/2} \ln(4\cos x)\ \mathrm{d}x \\ &= \int_0^{\pi/2} \ln(16\sin x\cos x)\ \mathrm{d}x \\ &= \int_0^{\pi/2} \ln(16\sin x\cos x)\ \mathrm{d}x \\ &= \int_0^{\pi/2} \ln(2) + \ln(8\sin x\cos x)\ \mathrm{d}x \\ &= \frac{\pi\ln 2}{2} + \int_0^{\pi/2} \ln(4\sin 2x)\ \mathrm{d}x \end{aligned}$$

Making the subsitution $u = 2x$ and noting that $\sin x$ assumes the same values from $[\pi/2, \pi]$ as $[0, \pi/2]$, we have:

$$\begin{aligned} 2I &= \frac{\pi\ln 2}{2} + \frac{1}{2}\int_0^{\pi} \ln(4\sin u)\ \mathrm{d}u \\ &= \frac{\pi\ln 2}{2} + \frac{1}{2}\left(\int_0^{\pi/2} \ln(4\sin u)\ \mathrm{d}u + \int_{\pi/2}^{\pi} \ln(4\sin u)\ \mathrm{d}u\right) \\ &= \frac{\pi\ln 2}{2} + \frac{2I}{2} \end{aligned}$$

Hence we have $I = \boxed{\dfrac{\pi\ln 2}{2}}$

[1] There was a typo in the actual test, $1/n^3$ should have been $1/n^2$