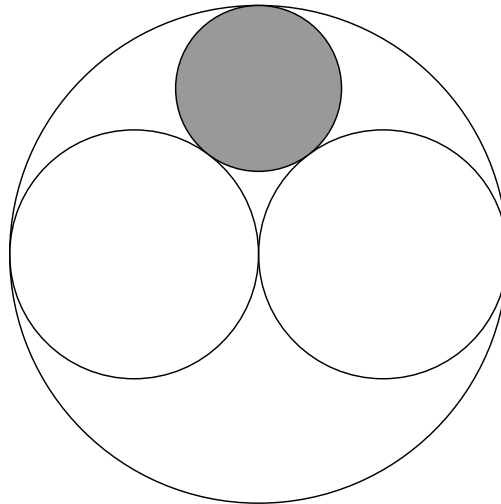


1. Frankie the frog likes to hop. On his first hop, he hops 1 meter. On each successive hop, he hops twice as far as he did on the previous hop. For example, on his second hop, he hops 2 meters, and on his third hop, he hops 4 meters. How many meters, in total, has he travelled after 6 hops?
2. Anton flips 5 fair coins. The probability that he gets an odd number of heads can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n . Compute $m + n$.
3. April discovers that the quadratic polynomial $x^2 + 5x + 3$ has distinct roots a and b . She also discovers that the quadratic polynomial $x^2 + 7x + 4$ has distinct roots c and d . Compute

$$ac + bc + bd + ad + a + b.$$

4. A rectangular picture frame that has a 2 inch border can exactly fit a 10 by 7 inch photo. What is the total area of the frame's border around the photo, in square inches?
5. Compute the median of the positive divisors of 9999.
6. Kaity only eats bread, pizza, and salad for her meals. However, she will refuse to have salad if she had pizza for the meal right before. Given that she eats 3 meals a day (not necessarily distinct), in how many ways can we arrange her meals for the day?
7. A triangle has side lengths 3, 4, and x , and another triangle has side lengths 3, 4, and $2x$. Assuming both triangles have positive area, compute the number of possible integer values for x .
8. In the diagram below, the largest circle has radius 30 and the other two white circles each have a radius of 15. Compute the radius of the shaded circle.



9. What is the remainder when 2022 is divided by 9?
 10. For how many positive integers x less than 2022 is $x^3 - x^2 + x - 1$ prime?
 11. A sphere and cylinder have the same volume, and both have radius 10. The height of the cylinder can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n . Compute $m + n$.
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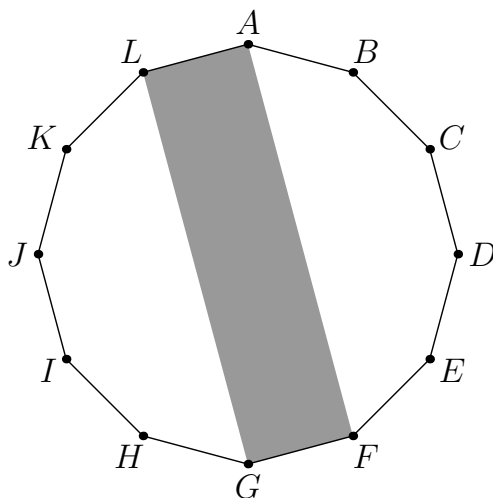
12. Amanda, Brianna, Chad, and Derrick are playing a game where they pass around a red flag. Two players “interact” whenever one passes the flag to the other. How many different ways can the flag be passed among the players such that

- (1) each pair of players interacts exactly once, and
- (2) Amanda both starts and ends the game with the flag?

13. Compute the value of

$$\frac{12}{1 + \frac{12}{1 + \frac{12}{1 + \dots}}}$$

14. Compute the sum of all positive integers a such that $a^2 - 505$ is a perfect square.
15. Alissa, Billy, Charles, Donovan, Eli, Faith, and Gerry each ask Sara a question. Sara must answer exactly 5 of them, and must choose an order in which to answer the questions. Furthermore, Sara must answer Alissa and Billy’s questions. In how many ways can Sara complete this task?
16. The integers $-x$, $x^2 - 1$, and x^3 form a non-decreasing arithmetic sequence (in that order). Compute the sum of all possible values of x^3 .
17. Moor and his 3 other friends are trying to split burgers equally, but they will have 2 left over. If they find another friend to split the burgers with, everyone can get an equal amount. What is the fewest number of burgers that Moor and his friends could have started with?
18. Consider regular dodecagon $ABCDEFGHIJKL$ below. The ratio of the area of rectangle $AFGL$ to the area of the dodecagon can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n . Compute $m + n$.



19. Compute the remainder when $3^{4^{5^6}}$ is divided by 4.
20. Fred is located at the middle of a 9 by 11 lattice (diagram below). At every second, he randomly moves to a neighboring point (left, right, up, or down), each with probability $1/4$. The probability that he is back at the middle after exactly 4 seconds can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n . Compute $m + n$.
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