**Time limit:** 30 minutes.
**Instructions:** For this test, you work in teams of five to solve 20 short answer questions. All answers on this test are integers. Please enter your answers as integers with no units or other symbols.
**No calculators.**

1. What is the area of a triangle with side lengths 6, 8, and 10?

2. Let $f(n) = \sqrt{n}$. If $f(f(f(n))) = 2$, compute $n$.

3. Anton is buying AguaFina water bottles. Each bottle costs 14 dollars, and Anton buys at least one water bottle. The number of dollars that Anton spends on AguaFina water bottles is a multiple of 10. What is the least number of water bottles he can buy?

4. Alex flips 3 fair coins in a row. The probability that the first and last flips are the same can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$. Compute $m + n$.

5. How many prime numbers $p$ satisfy the property that $p^2 - 1$ is *not* a multiple of 6?

6. In right triangle $\triangle ABC$ with $AB = 5$, $BC = 12$, and $CA = 13$, point $D$ lies on $\overline{CA}$ such that $AD = BD$. The length of $CD$ can then be expressed in the form $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$. Compute $m + n$.

7. Vivienne is deciding on what courses to take for Spring 2021, and she must choose from four math courses, three computer science courses, and five English courses. Vivienne decides that she will take one English course and two additional courses that are either computer science or math, and the order of the courses does not matter. How many choices does Vivienne have?

8. Square $ABCD$ has side length 2. Square $ACEF$ is drawn such that $B$ lies inside square $ACEF$. Compute the area of pentagon $AFECD$.

9. At the Boba Math Tournament, the Blackberry Milk Team has answered 4 out of the first 10 questions on the Boba Round correctly. If they answer all $p$ remaining questions correctly, they will have answered exactly $\frac{9p}{5}\%$ of the questions correctly in total. How many questions are on the Boba Round?

10. The sum of two positive integers is 2021 less than their product. If one of them is a perfect square, compute the sum of the two numbers.

11. Points $E$ and $F$ lie on edges $\overline{BC}$ and $\overline{DA}$ of unit square $ABCD$, respectively, such that $BE = \frac{1}{3}$ and $DF = \frac{1}{3}$. Line segments $\overline{AE}$ and $\overline{BF}$ intersect at point $G$. The area of triangle $EFG$ can be written in the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Compute $m+n$.

12. Compute the number of positive integers $n \leq 2020$ for which $n^{k+1}$ is a factor of $(1+2+3+\cdots+n)^k$ for some positive integer $k$.

13. How many permutations of 123456 are divisible by their last digit? For instance, 123456 is divisible by 6, but 561234 is not divisible by 4.

14. Compute the sum of all possible integer values for $n$ such that $n^2 - 2n - 120$ is a positive prime number.

15. Triangle $\triangle ABC$ has $AB = \sqrt{10}$, $BC = \sqrt{17}$, and $CA = \sqrt{41}$. The area of $\triangle ABC$ can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$. Compute $m + n$.

16. Let
$$f(x) = \frac{1 + x^3 + x^{10}}{1 + x^{10}}.$$
Compute
$$f(-20) + f(-19) + f(-18) + \cdots + f(20).$$

17. Leanne and Jing Jing are walking around the $xy$-plane. In one step, Leanne can move from any point $(x, y)$ to $(x + 1, y)$ or $(x, y + 1)$ and Jing Jing can move from $(x, y)$ to $(x - 2, y + 5)$ or $(x + 3, y - 1)$. The number of ways that Leanne can move from $(0, 0)$ to $(20, 20)$ is equal to the number of ways that Jing Jing can move from $(0, 0)$ to $(a, b)$, where $a$ and $b$ are positive integers. Compute the minimum possible value of $a + b$.

18. Compute the number of positive integers $1 < k < 2021$ such that the equation
$$x + \sqrt{kx} = kx + \sqrt{x}$$
has a positive rational solution for $x$.

19. In triangle $\triangle ABC$, point $D$ lies on $\overline{BC}$ with $\overline{AD} \perp \overline{BC}$. If $BD = 3AD$, and the area of $\triangle ABC$ is 15, then the minimum value of $AC^2$ is of the form $p\sqrt{q} - r$, where $p$, $q$, and $r$ are positive integers and $q$ is not divisible by the square of any prime number. Compute $p + q + r$.

20. Suppose the decimal representation of $\frac{1}{n}$ is in the form $0.p_1p_2\ldots p_j\overline{d_1d_2\ldots d_k}$, where $p_1, \ldots, p_j, d_1, \ldots, d_k$ are decimal digits, and $j$ and $k$ are the smallest possible nonnegative integers (i.e. it's possible for $j = 0$ or $k = 0$). We define the *preperiod* of $\frac{1}{n}$ to be $j$ and the *period* of $\frac{1}{n}$ to be $k$. For example, $\frac{1}{6} = 0.16666\ldots$ has preperiod 1 and period 1, $\frac{1}{7} = 0.\overline{142857}$ has preperiod 0 and period 6, and $\frac{1}{4} = 0.25$ has preperiod 2 and period 0. What is the smallest positive integer $n$ such that the sum of the preperiod and period of $\frac{1}{n}$ is 8?