Time limit: 10 minutes.
Instructions: For this test, you work in teams of five to solve 50 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. True or False? The answer to the question is true.

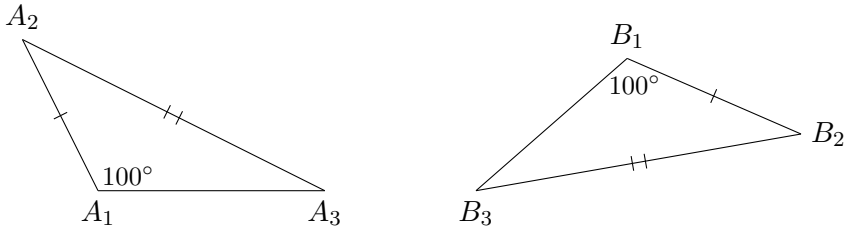
2. True or False? If p is a prime, m is an integer, and $\frac{m}{p}$ is an integer, then $p = m$.

3. True or False? If a, b, c, and d are nonzero real numbers where $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{b} = \frac{a+c}{b+d} = \frac{c}{d}$.

4. True or False? If $A_1A_2A_3$ and $B_1B_2B_3$ are triangles in which

$$\angle A_2A_1A_3 = \angle B_2B_1B_3 = 100^\circ$$
$$A_1A_2 = B_1B_2$$
$$A_2A_3 = B_2B_3$$

as shown, then triangle $A_1A_2A_3$ is congruent to triangle $B_1B_2B_3$.



5. True or False? If three fair coins are flipped, then the probability of landing two heads and one tails in any order is greater than the probability of landing three heads.

6. For a real number x, the *floor* of x is denoted $\lfloor x \rfloor$, and defined to be the largest integer less than or equal to x. For example, $\lfloor 3.2 \rfloor = 3$ and $\lfloor -1.5 \rfloor = -2$.

   True or False? If x and y are real numbers, then

$$\left\lfloor \frac{\lfloor x \rfloor}{y} \right\rfloor = \left\lfloor \frac{x}{y} \right\rfloor.$$

7. True or False? If $ABCD$ is a quadrilateral, then the perpendicular bisectors of $ABCD$ all intersect in one point.

8. True or False? All positive perfect cubes have an odd number of positive divisors.

9. True or False? $2017^{2018} > 2018^{2017}$.

10. True or False? A parallelogram with vertices at $(0, 0)$, (a_1, a_2), (b_1, b_2), and $(a_1 + b_1, a_2 + b_2)$ has area $|a_1b_2 - b_1a_2|$.

11. Compute the product of 9 and 17.

12. What is the smallest positive integer divisible by $1, 2, 3, 4, 5, 6$, and 7?

13. How many integer solutions are there to the equation $x^3 = 10$?

14. If the complement of an angle is $49.75°$, what is the supplement of that angle in degrees? Express your answer as a decimal or a fraction in lowest terms.

15. What is the maximum number of 30×30 squares that can be placed in a 100×100 square such that no two of the 30×30 squares overlap and each edge of the 30×30 squares is parallel to an edge of the 100×100 square?

16. A medium T-shirt is 10% bigger than a small one, and a large T-shirt is 10% bigger than a medium one. What percent larger is a large T-shirt than a small one?

17. Compute $85 \cdot 93 - 81 \cdot 97$.

18. Compute 115^2.

19. How many ways can three runners place in a race if ties are possible?

20. At Abe's Pizza, there are 4 choices of toppings, 3 choices of cheese, and 3 choices of crust. Each pizza has exactly 1 topping, 1 cheese, and 1 crust. How many different pizzas can be ordered at Abe's Pizza?

21. Let M be the set of the first 200 positive odd integers. Let N be the sum of the first 100 positive even integers. Find $M - N$.

22. What is the time 500 minutes after 10:30 AM? Make sure to specify whether your answer is AM or PM.

23. Compute $6 - 3 + \frac{3}{2} - \frac{3}{4} + \frac{3}{8} - \cdots$.

24. If a and b are two positive prime numbers whose squares sum to 173, what is the product of a and b?

25. What is the least nonnegative integer n such that $2^{2^n} + 3$ is composite?

26. How many positive factors does 6! have?

27. Compute $37^3 + 3 \cdot 37^2 \cdot 63 + 3 \cdot 37 \cdot 63^2 + 63^3$.

28. What is the largest prime number that is a factor of 2018?

29. The greatest common factor of two integers a and b is 22. The least common multiple of a and b is 88. What is the product of a and b?

30. If the chairs in an auditorium are organized into equal-length rows of 13 chairs, there are 11 left over. If they are organized into equal-length rows of 7 chairs, there are 5 left over. What is the smallest number of chairs that the auditorium could have?

31. What is the maximum area of a rectangle with perimeter 100?

32. What is the area of a circle with circumference 12π?

33. What is the angle, in degrees, formed by the minor arc of the hour and minute hand of a clock at 11:40?

34. Find the area of an equilateral triangle with side length 10.

35. Compute $\frac{1}{2\cdot3} + \frac{1}{3\cdot4} + ... + \frac{1}{99\cdot100}$. Express your answer as a decimal or a fraction in lowest terms.

36. For how many integers n is $n^2 + 11n + 30$ a prime number?

37. Call a number *special* if it has exactly three prime factors. What is the smallest special number over 100? (Note: 1 is not prime.)

38. The class average for a math exam was 88 out of 100. After a student with a score of 60 leaves the class, the average rises to 95. How many students are now in the class?

39. How many diagonals does a regular decagon have? (A diagonal is any segment connecting two vertices that do not share a side, and a decagon has 10 vertices.)

40. What is the smallest positive integer with 8 positive divisors?

41. A math club with 7 girls and 5 boys wants to select a team of 3 girls and 2 boys for BmMT. How many different teams can the math club make?

42. How many pairs of positive integers a and b exist that satisfy $a + b = 100$ and $a < b$?

43. 10 distinct circles are drawn in the plane. What is the maximum number of points on an intersection between two or more circles?

44. How many prime numbers are there between 50 and 100?

45. All diagonals in a regular hexagon are drawn. How many points inside the hexagon are the intersection of two or more diagonals? (A diagonal is any segment connecting two vertices that do not share a side.)

46. Express $0.5\overline{72}$ as a fraction in lowest terms.

47. How many ways are there to make 88 cents using only pennies, nickels, and dimes? (Pennies are worth 1 cent, nickels are worth 5 cents, and dimes are worth 10 cents.)

48. A 6-sided die is rolled 3 times. What is the probability that the product of the rolls is prime? Express your answer as a decimal or a fraction in lowest terms.

49. BmMT needs to buy breakfast for its staff. The local bagel shop sells bags of 12 bagels. If each bagel can be either a plain bagel, sesame bagel, or garlic bagel, how many different bags of bagels can BmMT buy?

50. How many positive divisors of 10! are perfect cubes?